DWARKA INTERNATIONAL SCHOOL

MATHEMATICS

Class-XII

UNIT-I. RELATIONS & FUNCTIONS

1. Define relation R on the set **N** of natural numbers by R={ (x,y): $y= x+7$, x is a natural Number less than 4; $x, y \in$ **N** }.Write down the domain and the range.

2. Let R be the relation in the set N given by R = {(a,b); a = b – 2 , b > 6} Whether the relation is reflexive or not ? Justify your answer.

3. Let R be the relation on R defined as (a , b) $\in$ R iff 1+ ab > 0 $\forall$ a,b $\in$R.
   a. Show that R is symmetric.
   b. Show that R is reflexive.
   c. Show that R is not transitive.

4. Let A = {-1, 0, 1} and B = {0 , 1}. State whether the function f: A → B defined by $f(x) = x^2$ is bijective .

5. Show the function f: R→R defined by f(x) = $\frac{2x-1}{3}$, x∈ R is one-one and onto function. Also find the inverse of the function f.

6. Show that the relation R on A ,A = { x| x $\in$Ƶ , 0 ≤ x ≤ 12 } ,
R = {(a ,b): |a - b| is multiple of 3.} is an equivalence relation.

7. Let A = Set of all triangles in a plane and R is defined by
R={(T_1,T_2) : $T_1,T_2 \in$ A & $T_1 \sim T_2$ }. Show that the R is equivalence relation. Consider the right angled Δs, T_1 with size 3,4,5; T_2 with size 5, 12,13; T_3 with side 6, 8, 10; Which of the pairs are related?

8. Consider a function f :R_+à[-5, ∞) defined $f(x) = 9x^2 + 6x - 5$. Show that f is bijective function.

9.Prove the following: $2\,tan^{-1}\left(\frac{1}{3}\right) + tan^{-1}\left(\frac{1}{7}\right) = \frac{\pi}{4}$

10.Prove the following: $\boldsymbol{tan^{-1}\left(\frac{1}{4}\right) + tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)}$

11.Prove that: $\boldsymbol{\tan^{-1}\left(\frac{62}{16}\right) = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}}$.

12. $tan^{-1}\left(\frac{1}{3}\right) + tan^{-1}\left(\frac{1}{5}\right) + tan^{-1}\left(\frac{1}{7}\right) + tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$.

13.Prove the following : $cot^{-1}\left(\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-s}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$.

14. Solve for x : $2tan^{-1}(\cos x) = tan^{-1}(2\;cosec\;x)$.

15. Solve for x: $tan^{-1}(2x) + tan^{-1}(3x) = \frac{\pi}{4}$, x>0.

16. Solve for x: $\boldsymbol{tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}}$.

17. Solve for x: $tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{2\pi}{3}$

18. Let N be the set of all natural numbers & R be the relation on N × N defined by { (a,b) R (c,d) iff a + d = b + c}. Show that R is an equivalence relation.
19. Let A= {1,2,3,…….,9} and R be the relation in A×A defined by (a,b) R(c,d) iff a+d=b+c for (a,b),(c,d)$\in A \times A$. Prove that R is an equivalence relation and also find the equivalence class $|(2,5)|$.
20. Let N denote the set of all natural numbers and R be the relation on NXN defined by $(a,b)R(c,d) \Leftrightarrow ad(b+c) = bc(a+d)$ prove that R is an equivalence relation on NXN.

UNIT-II. ALGEBRA

1. If A=$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, prove that $A^2 - 4A - 5I = 0$. Hence find A^{-1}

2. A=$\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and I=$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$.

3. Find the matrix P satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

4. Let A=$\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Prove that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.

5. Express the matrix A=$\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$ as the sum of a symmetric and skew –symmetric matrix.
6. Using matrix method, solve the system: x + y + z = 6 ; y + 3z = 11, x – 2y + z = 0
7. Using matrix method, solve the system: 3x -2 y +3 z = 8 ; 2x + y – z = 1 ; 4x –3 y +2 z = 4
8. Solve the following system of equation by matrix method, where x# 0, y # 0, z # 0;

$\boxed{\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10}$; $\boxed{\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10}$; $\boxed{\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13}$

9. Find the product AB, where A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and use it to solve the equations x – y = 3, 2x + 3y + 4z = 17, y + 2z = 7

UNIT-III. CALCULUS

1. If f(x)= $\begin{cases} ax+1, & if\ x \le 3 \\ bx+3, & if\ x > 3 \end{cases}$ is continuous at x=3. Find the values of a add b.

2. f(x)=$\begin{cases} k(x^2-2x), & x \le 0 \\ 4x+1, & x > 0 \end{cases}$ is continuous at x=0. Find the value of k also discuss about continuity at x=1.

3. f(x)= $\begin{cases} 5, & if\ x \le 2 \\ ax+b, & 2 < x < 10 \\ 21, & if\ x \ge 10 \end{cases}$ is continuous function.

4. Show that f(x)= $|x-3|$ is continuous but not differentiable at x=3

5. f(x)= $\begin{cases} \frac{e^x-1}{x}, & if\ x \neq 0 \\ k, & if\ x = 0 \end{cases}$ is continuous at x=0

6. x=$\frac{sin^3 t}{\sqrt{cos2t}}$, y=$\frac{cos^3 t}{\sqrt{cos2t}}$. Find $\frac{d^2y}{dx^2}$.

7. if $x\sqrt{1+y} + y\sqrt{1+x}$=0 then prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$

8. if $(x-a)^2 + (y-b)^2 = c^2$ then prove that $\frac{[1+(\frac{dy}{dx})^2]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is independent of a and b

9. If cos y = x cos(a+y) then prove that $\frac{dy}{dx} = \frac{cos^2(a+y)}{\sin a}$

10. If y = $e^{acos^{-1}x}$ then show that (1-x^2)$\frac{d^2y}{dx^2}$ - x$\frac{dy}{dx}$- a^2y = 0

11. If y = a sin x + b cos x, then prove that $\frac{d^2y}{dx^2}$ + y = 0.

12. If y = 500e^{7x} + 600e^{-7x}, prove that $\frac{d^2y}{dx^2}$ - 49y = 0.

13. Find the second order derivative of x^3 + log x.

14. If y = $\tan^{-1} x$, prove that (1 + x^2) y_2 + 2x y_1 = 0.

15. Find the intervals in which the function f given by f(x) = -2x^3 - 9 x^2- 12x +1 is strictly decreasing .

16. Find the intervals in which the function f given by f(x) = $\sin x$+ $\cos x$, 0$\leq$ x$\leq$ 2π is strictly increasing .

17. Find the value of x for which y = $[\,x\,(\,x-2\,)]^2$ is an increasing function.

18. Prove that the function given by f(x) = cos x is
    (a) strictly decreasing in (0,π).
    (b) strictly increasing in (π, 2 π).
    (c) neither increasing nor decreasing in (0, 2 π).

19. The volume of a cube is increasing at the rate 10 $\frac{cm^3}{sec}$. How fast is the surface area increasing when the length of an edge is 15 cm?

20. A particle move along the curve 6y=x^3+2. Find the points on the curve at which the Y coordinate is changing 8 times as fast as the X –coordinate.

21. The radius of an air buble is increasing at the rate of $\frac{1}{2}$ cm/s. At what rate is the volume of the buble increasing when the radius is 1cm?

22. Sand is pouring from a pipe at the rate of 12$\frac{cm^3}{sec}$.The falling sand forms, a cone on the ground in such a way that the hight of the cone is always one –sixth of the radius of the base .How fast is the height of the sand cone increasing when the height is 4cm?

23. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere.

24. A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximumpossible.

25. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top by cutting off squares from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is the maximum possible.?

26. Show that the height of cylinder of maximum volume that can be inscribed in a sphere of radius ‘a’ is $\boxed{\frac{2a}{\sqrt{3}}}$.

27. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\mathrm{Sin}^{-1}(1/3)$.

28. Evaluate:$I = \int \frac{1-x^2}{x(1-2x)} dx$

29. Evaluate:$\int \frac{(5x+3)dx}{\sqrt{x^2+4x+10}}$

30. Evaluate: $\int \frac{dx}{x^2-6x+13}$

31. Evaluate: $\mathrm{I} = \int_0^{\frac{\pi}{2}} \frac{dx}{1+\sqrt{cotx}}$ OR $\mathrm{I} = \int_0^{\frac{\pi}{2}} \frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}} \mathrm{dx}$

32. Evaluate:$\int \frac{dx}{\sqrt{x^2+8x+10}}$

33. Evaluate: $I = \int \frac{1}{1+e^x} dx$

34. Evaluate:$I = \int \frac{dx}{(x^2+1)(x^2+2)}$

35. Find area of region bounded by y=x^2 and y=$|x|$.

36. Solve the differential equation: $x^2 \frac{dy}{dx} = 2xy + y^2$.

37. Solve the differential equation: $\frac{dy}{dx} - \frac{y}{x} + cosec\left(\frac{y}{x}\right) = 0; y = 0\ when\ x = 1.$

38. Solve the given differential equation $\frac{dy}{dx}$ = $(1+x^2)(1+y^2)$.

39. Solve $e^{\frac{dy}{dx}} = 2$.

40. Using integration find the area of the region bounded by the parabola $y^2=8x$ and latus rectum.

41. Find the area of the region: $\{(x, y): x^2 + y^2 \leq 1 \leq x + y\}$,using integration

42. Find the area of the region in the first quadrant enclosed by the x-axis, the line y = x and the circle $x^2 + y^2 = 32$ using integration

43. Solve :$(3x^2 + y)\frac{dx}{dy} = x, x > 0, when x = 1, y = 1.$

44. Solve :$xdy + (y - x^3)dx = 0$

45. Solve the differential equation:$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$

UNIT-IV. VECTORS AND THREE DIMENSIONAL GEOMETRY

1. Show that the vectors are $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$ and $\vec{b} = 6\hat{\imath} - \hat{\jmath} + 4\hat{k}$ are orthogonal.

2. Show that the vectors $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$ and $\vec{b} = 4\hat{\imath} + 6\hat{\jmath} + 2\hat{k}$ are orthogonal.

3. If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$, $\vec{b} = 5\hat{\imath} + 4\hat{\jmath} - \hat{k}$, $\vec{c} = 3\hat{\imath} + 6\hat{\jmath} + 2\hat{k}$ and $\vec{d} = \hat{\imath} + 2\hat{\jmath}$, show that $\vec{b} - \vec{a}$ is perpendicular to $\vec{d} - \vec{c}$.

4. Find the angle between the vectors $\vec{a} = 3\hat{\imath} + 2\,\hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} - \hat{\jmath} - 4\,\hat{k}$ using the scalar product of the vectors.

5. If $\vec{a} = 2i + 2j + 3k$, $\vec{b} = -i + 2j + k$ $and \vec{c}$ = 3i+j are such that $\vec{a} + \mu\vec{b}$ is perpendicular to $\vec{c}$, then find the value of μ .

6. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ .Find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.

7. If $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|$= 3, $|\vec{b}|$= 4 and $|\vec{c}|$= 5 and each one of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$.

8. Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\mu = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$, if $|\vec{a}|$= 1, $|\vec{b}|$=4 and $|\vec{c}|$= 2.

9. Find the shortest distance between the lines whose vector equations are
$\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - 3\hat{\jmath} - 2\hat{k})$ and $\vec{r} = (4\hat{\imath} + 5\hat{\jmath} + 6\hat{k}) + \mu(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$

10. Find the shortest distance between the two lines $\boxed{\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})}$ and $\boxed{\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})}$.

11. Find the foot of perpendicular and image of the point P(1, 2, 4) on the $\vec{r} = (4\hat{\imath} + 5\hat{\jmath} + 6\hat{k}) + \mu(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$

UNIT-V. LINEAR PROGRAMMING

1. A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 labour hours for fabricating and 2 labour hours for finishing. Each piece of model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many pieces of model A and Model B should be manufactured per week to realize a maximum profit? After income tax, sale tax, service tax, the net profit of the company is Rs 1,14,000. Should owner of the company close down the company? Discuss briefly.

2. A dealer in rural area wishes to purchase a number of sewing machine. He has only Rs. 5760 to invest and has space for at most 20 items. An electronic sewing machine costs him Rs. 360 and a manually operated sewing machine Rs. 240. He can sell an electronic machine at a profit of Rs. 22 and manually operated machine at a profit of Rs. 18. Assuming that he can sell all the items that he can buy how should he invest his money in order to maximize his profit. Make it as a linear programming problem and solve it graphically. Keeping the rural background justify the values to be promoted for the selection of manually operated machine.

3. A manufacturer manufactures two types of steel trunks. He has two machines A and B. For completing the first type of trunk, it requires 3 hours on machine A and 1 hour on machine B; whereas the second type of trunk requires 3 hours on machine A and two hours on machine B. Machine A can work for 18 hours and B for 8 hours only per day. There is a profit of Rs.30 on the first type of the trunk and Rs.48 on second type per trunk. How many trunk of each type should be manufactured every day to earn maximum profit? Solve the problem graphically.

4. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods A and B are available at a cost of Rs.5 and Rs.4 per unit respectively. One unit of the food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories, while one unit of the food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the foods A and B should be used to have least cost, but it must satisfy the requirements of the sick person. Form the question as LPP and solve it graphically. Explain the importance of balanced diet.

UNIT-VI. PROBABILITY

1. A fair die tossed thrice. Find the probability of getting an odd number at least once.

2. Probability of solving specific problem independently by A and B are 1/2 and 1/3 respectively. If both try to solve the problem independently, find the probability that (1) the problem is solved (2) exactly one of them solved the problem

3. Event A and B are such that P (A) = 1/2, P (B) =7/12 and P(not A or not B) =1/4. State whether A and B are independent.

4. Two cards are drawn at random and without replacement from a pack of 52 cards. Find the probability that both the cards are black.

5. The probabilities of A, B and C hitting a target are 1/3,2/7 and 3/8 respectively. If all the three tried to shoot the target simultaneously, find the probability that exactly one of them can shoot it.

6. A husband and a wife appear for interview for two vacancies for the same post. The probability of husband's selection is 1/7 and that of wife's is 1/5.What is the probability that (1) only one of them will be selected (2) at least one of them will be selected.

7. A coin is tossed once. If it shows a head,it is tossed again but if it shows a tail,then a die is thrown. If all possible outcomes are equally likely, find the probability that the die shows a number greater than 4, if it is known that the first throw of coin results a tail.

8. Three persons A B and C fire a target in turn,starting with A. Their probability of fitting the target are 0.5,0.3 and o.2 respectively. Find the probability of at most one hit.

9. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that (1) both balls are red (2) one of them is black and other is red.

10. In an examination, an examinee either guesses or copies or knows the answer of multiple choice questions with four choices. The probability that he makes a guess is $\frac{1}{3}$ and probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copied it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question, given that he correctly answered it.

11. A problem in statistics is given to three students A, B and C, whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved.

CHAPTER INTEGRALS AND AREA BOUNDED UNDER THE CURVES

1. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

2. Show that $\int_0^{\pi/2} \sqrt{\tan x} + \sqrt{\cot x} = \sqrt{2}\pi$

3. Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

4. Evaluate : $\int \frac{e^x}{\sqrt{5-4e^x - e^{2x}}} dx$

5. Evaluate : $\int \frac{(x-4)e^x}{(x-2)^3} dx$

6. Evaluate: $\int_0^{\pi/2} (2 \log \sin x - \log \sin 2x)\, dx$.

7. Evaluate : $\int \frac{dx}{\sqrt{5-4x-2x^2}}$

8. Evaluate : $\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2 \sin^2 x}$

9. Evaluate: $\int \frac{\cos x}{(2+\sin x)(3+4\sin x)}\,dx$

10. Evaluate: $\int x^2 . \cos^{-1} x \, dx$

11. Evaluate: $\int \frac{x^4\,dx}{(x-1)(x^2+1)}$

12. Evaluate: $\int_1^4 [|x-1|+|x-2|+|x-4|]dx$

13. Evaluate:

$$\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\,dx$$

14. Evaluate : $\int e^x \left(\frac{\sin 4x - 4}{1-\cos 4x}\right) dx.$

15. Evaluate : $\int \frac{1-x^2}{x(1-2x)}\,dx.$

16. Evaluate : $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\,dx.$

17. Using integration, find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$

18. Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\,dx.$

19. Evaluate: $\int_0^{\pi} \frac{x}{1+\sin x}\,dx.$